Decompositions of Hypergraphs

Felix Joos

July 2020

UNIVERSITÄT
HEIDELBERG
ZUKUNFT
SEIT 1386

Old results

Theorem (Walecki, 1892)

K_{2n+1} has a decomposition into edge-disjoint Hamilton cycles.

Old results

Theorem (Walecki, 1892)

K_{2n+1} has a decomposition into edge-disjoint Hamilton cycles.

Theorem (Dirac, 1952)

Every graph G on $n \geq 3$ vertices with $\delta(G) \geq \frac{n}{2}$ contains a Hamilton cycle.

Old results

Theorem (Walecki, 1892)

K_{2n+1} has a decomposition into edge-disjoint Hamilton cycles.

Theorem (Dirac, 1952)

Every graph G on $n \geq 3$ vertices with $\delta(G) \geq \frac{n}{2}$ contains a Hamilton cycle.

Theorem (Hajnal, Szemerédi, 1970)

$k|n$ and $\delta(G) \geq \frac{k-1}{k}n$, then G contains a K_k-factor.

Generalizations

Theorem (Csaba, Kühn, Lo, Osthus, Treglown, 2016)

G r-regular with $r \geq \frac{n}{2}$ and even, n large, then G has a Hamilton decomposition.

Generalizations

Theorem (Csaba, Kühn, Lo, Osthus, Treglown, 2016)

G r-regular with $r \geq \frac{n}{2}$ and even, n large, then G has a Hamilton decomposition.

Theorem (Böttcher, Schacht, Taraz, 2009)

$\chi(H) = k$, H has bandwidth $o(n)$ and $\Delta(H) = O(1)$
$\delta(G) \geq (\frac{k-1}{k} + o(1))n$, then $H \subset G$.

Open problems

Conjecture (Nash-Williams, 1970)

$\delta(G) \geq \frac{3n}{4}$ and G is triangle-divisible, then G has a triangle decomposition

Open problems

Conjecture (Nash-Williams, 1970)

$\delta(G) \geq \frac{3n}{4}$ and G is triangle-divisible, then G has a triangle decomposition

Problem

Given a graph H, determine δ_H where δ_H is the least δ such that for every $\varepsilon > 0$ and G on n (large) vertices with $\delta(G) \geq (\delta + \varepsilon)n$ has a H-decomposition subject to divisibility conditions?

A step forward

Fractional H-decomposition of G:
ω: $\{$copies of H in $G\} \to [0,1]$ such that $\sum_{H \ni e} \omega(H) = 1$ for $e \in E(G)$.

A step forward

Fractional H-decomposition of G:
ω: $\{$copies of H in $G\} \to [0,1]$ such that $\sum_{H \ni e} \omega(H) = 1$ for $e \in E(G)$.

δ^*_H fractional version of δ_H.

A step forward

Fractional H-decomposition of G:
ω: $\{$copies of H in $G\} \to [0,1]$ such that $\sum_{H \ni e} \omega(H) = 1$ for $e \in E(G)$.

δ^*_H fractional version of δ_H.

Theorem (Barber, Kühn, Lo, Osthus, 2016; Glock, Kühn, Lo, Montgomery, Osthus, 2019)

$\delta_H \in \{\delta^*_H, 1 - \frac{1}{\chi}, 1 - \frac{1}{\chi+1}\}$ *for* $\chi = \chi(H) \geq 5$
solved for bipartite H
General tool: turning fractional decompositions into decompositions

Cycles

Theorem (Barber, Kühn, Lo, Osthus, 2016)

$\delta_{C_4} = \frac{2}{3}$ *and* $\delta_{C_{2\ell}} = \frac{1}{2}$ *for* $\ell \geq 3$

$\delta_{C_{2\ell+1}} = \delta^*_{C_{2\ell+1}}$

Cycles

Theorem (Barber, Kühn, Lo, Osthus, 2016)

$\delta_{C_4} = \frac{2}{3}$ *and* $\delta_{C_{2\ell}} = \frac{1}{2}$ *for* $\ell \geq 3$

$\delta_{C_{2\ell+1}} = \delta^*_{C_{2\ell+1}}$

Theorem (J., M. Kühn, 2020^+)

$\delta_{C_{2\ell+1}} \to \frac{1}{2}$ $(\ell \to \infty)$

Hypergraphs - old results

G k-uniform (k-graph): edges of size k
$d_m(S) =$ number of edges containing S for $|S| = m$
$\delta_m(G) = \min_S d_m(S)$
$\delta(G) = \delta_{k-1}(G)$

Hypergraphs - old results

G k-uniform (k-graph): edges of size k
$d_m(S) =$ number of edges containing S for $|S| = m$
$\delta_m(G) = \min_S d_m(S)$
$\delta(G) = \delta_{k-1}(G)$

tight cycle = cyclic vertex ordering, all k consecutive vertices form an edge

Hypergraphs - old results

G k-uniform (k-graph): edges of size k
$d_m(S) =$ number of edges containing S for $|S| = m$
$\delta_m(G) = \min_S d_m(S)$
$\delta(G) = \delta_{k-1}(G)$

tight cycle = cyclic vertex ordering, all k consecutive vertices form an edge

Theorem (Rödl, Ruciński, Szemerédi, 2008)

$\delta(G) \geq (\frac{1}{2} + o(1))n$, then G contains a (tight) Hamilton cycle

Hypergraphs - old results

G k-uniform (k-graph): edges of size k
$d_m(S) =$ number of edges containing S for $|S| = m$
$\delta_m(G) = \min_S d_m(S)$
$\delta(G) = \delta_{k-1}(G)$

tight cycle = cyclic vertex ordering, all k consecutive vertices form an edge

Theorem (Rödl, Ruciński, Szemerédi, 2008)
$\delta(G) \geq (\frac{1}{2} + o(1))n$, then G contains a (tight) Hamilton cycle

Theorem (Lang, Sahueza-Matamala; Polcyn, Reiher, Rödl, Schülke, 2020^{+})
$\delta_{k-2}(G) \geq (\frac{5}{9} + o(1))n^2/2$, then G contains a (tight) Hamilton cycle

New results

New results

Theorem (J., Kühn, 2020^+)

$\delta^*_{C_\ell^{(k)}} \to \frac{1}{2}$ *for* $\ell \to \infty$

New results

Theorem (J., Kühn, 2020^{+})

$\delta^*_{C_\ell^{(k)}} \to \frac{1}{2}$ *for* $\ell \to \infty$

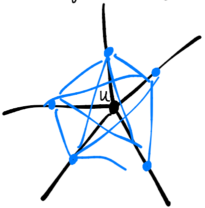
Proof method: Restriction systems + random walks

Proof Sketch

1 Simple random walk on a regular, non-bipartite, connected graph

$\Rightarrow$ Uniform limit distribution

2 Find a d-regular subgraph in the line graph



degree $> d$: delete some edges

restriction system

3 Markov chain on ordered edges (avoid walking around a vertex)

4 $\mathbb{P}[\text{from } \vec{e} \text{ to } \vec{f} \text{ in } k \text{ steps}] = p_k \Leftrightarrow \#\, k\text{-walks from } \vec{e} \text{ to } \vec{f} = p_k\, d^k$

conform with the restriction system

5 $p_k \xrightarrow[k\to\infty]{} \frac{1}{2e(G)}$ (very quickly in k)

ind. of e and f!

New results II

New results II

Theorem (Rödl, Ruciński, Szemerédi, 2008)

$\forall \epsilon > 0, k \in \mathbb{N}$ *the following holds for all large n:*
G k-graph with $\delta(G) \geq (\frac{1}{2} + \varepsilon)n$,
then G contains a (tight) Hamilton cycle

New results II

Theorem (Rödl, Ruciński, Szemerédi, 2008)

$\forall \epsilon > 0, k \in \mathbb{N}$ *the following holds for all large n:*
G k-graph with $\delta(G) \geq (\frac{1}{2} + \varepsilon)n$*,*
then G contains a (tight) Hamilton cycle

Theorem (J., Kühn, Schülke, 2020^+)

$\forall \epsilon > 0, k \in \mathbb{N}\ \exists\ \epsilon' > 0$ *such that for all large n:*

New results II

Theorem (Rödl, Ruciński, Szemerédi, 2008)

$\forall \epsilon > 0, k \in \mathbb{N}$ *the following holds for all large n:*
G k-graph with $\delta(G) \geq (\frac{1}{2} + \varepsilon)n$*,*
then G contains a (tight) Hamilton cycle

Theorem (J., Kühn, Schülke, 2020^{+})

$\forall \epsilon > 0, k \in \mathbb{N}\ \exists\ \epsilon' > 0$ *such that for all large n:*

- *G k-graph with* $\delta(G) \geq (\frac{1}{2} + \epsilon)n$

New results II

Theorem (Rödl, Ruciński, Szemerédi, 2008)

$\forall \epsilon > 0, k \in \mathbb{N}$ *the following holds for all large n:*
G k-graph with $\delta(G) \geq (\frac{1}{2} + \varepsilon)n$,
then G contains a (tight) Hamilton cycle

Theorem (J., Kühn, Schülke, 2020^{+})

$\forall \epsilon > 0, k \in \mathbb{N}\ \exists\ \epsilon' > 0$ *such that for all large n:*

- *G k-graph with* $\delta(G) \geq (\frac{1}{2} + \epsilon)n$
- $|d_1(v) - d_1(u)| \leq \epsilon' n^{k-1}$

New results II

Theorem (Rödl, Ruciński, Szemerédi, 2008)

$\forall \epsilon > 0, k \in \mathbb{N}$ *the following holds for all large n:*
G k-graph with $\delta(G) \geq (\frac{1}{2} + \varepsilon)n$,
then G contains a (tight) Hamilton cycle

Theorem (J., Kühn, Schülke, 2020^{+})

$\forall \epsilon > 0, k \in \mathbb{N} \; \exists \; \epsilon' > 0$ *such that for all large n:*
- *G k-graph with* $\delta(G) \geq (\frac{1}{2} + \epsilon)n$
- $|d_1(v) - d_1(u)| \leq \epsilon' n^{k-1}$

then G contains $(e(G) - \epsilon n^k)/n$ *edge-disjoint Hamilton cycles.*

New results II

Theorem (J., Kühn, Schülke, 2020^+)

$\forall \epsilon > 0, k \in \mathbb{N} \; \exists \; \epsilon' > 0$ *such that for all large n:*

- *G k-graph with* $\delta(G) \geq (\frac{1}{2} + \epsilon)n$
- $|d_1(v) - d_1(u)| \leq \epsilon' n^{k-1}$

then G contains $(e(G) - \epsilon n^k)/n$ *edge-disjoint Hamilton cycles.*

New results II

Theorem (J., Kühn, Schülke, 2020^+)

$\forall \epsilon > 0, k \in \mathbb{N}\ \exists\ \epsilon' > 0$ such that for all large n:
• G k-graph with $\delta(G) \geq (\frac{1}{2} + \epsilon)n$
• $|d_1(v) - d_1(u)| \leq \epsilon' n^{k-1}$
then G contains $(e(G) - \epsilon n^k)/n$ edge-disjoint Hamilton cycles.

Corollary

Vertex-regular k-graphs G with $\delta(G) \geq (\frac{1}{2} + o(1))n$ can be approximately decomposed into Hamilton cycles with an arbitrary good precision.

New results II

Theorem (J., Kühn, Schülke, 2020$^+$)

$\forall \epsilon > 0, k \in \mathbb{N}\ \exists\ \epsilon' > 0$ such that for all large n:

- *G k-graph with $\delta(G) \geq (\frac{1}{2} + \epsilon)n$*
- *$|d_1(v) - d_1(u)| \leq \epsilon' n^{k-1}$*

then G contains $(e(G) - \epsilon n^k)/n$ edge-disjoint Hamilton cycles.

Corollary

Vertex-regular k-graphs G with $\delta(G) \geq (\frac{1}{2} + o(1))n$ can be approximately decomposed into Hamilton cycles with an arbitrary good precision.

Proof method: $\delta^*_{C_\ell} \leq \frac{1}{2} + \epsilon$ for large enough ℓ; random process; absorption

Summary I

Summary I

- Fractional ℓ-cycle decompositions

Summary I

- Fractional ℓ-cycle decompositions
- approx. decompositions into Hamilton cycles

Summary I

- Fractional ℓ-cycle decompositions
- approx. decompositions into Hamilton cycles

in hypergraphs under very weak assumptions

Graph decompositions

Three conjectures:

- Ringel: K_{2n+1} into any tree with n edges
- Tree packing conj.: K_n into trees $T_1, \ldots, T_{n-1}$ with $e(T_i) = i$
- Oberwolfach problem: K_{2n+1} into any spanning union of cycles

Graph decompositions - progress

Approximate decompositions:

- $\Delta = O(1)$, trees, almost spanning, K_n Böttcher, Hladký, Piguet, Taraz, 16

Graph decompositions - progress

Approximate decompositions:

- $\Delta = O(1)$, trees, almost spanning, K_n Böttcher, Hladký, Piguet, Taraz, 16
- $\Delta = O(1)$, separable, almost spanning, K_n Messuti, Rödl, Schacht, 17

Graph decompositions - progress

Approximate decompositions:

- $\Delta = O(1)$, trees, almost spanning, K_n Böttcher, Hladký, Piguet, Taraz, 16
- $\Delta = O(1)$, separable, almost spanning, K_n Messuti, Rödl, Schacht, 17
- $\Delta = O(1)$, separable, spanning, K_n Ferber, Lee, Mousset, 16

Graph decompositions - progress

Approximate decompositions:

- $\Delta = O(1)$, trees, almost spanning, K_n Böttcher, Hladký, Piguet, Taraz, 16
- $\Delta = O(1)$, separable, almost spanning, K_n Messuti, Rödl, Schacht, 17
- $\Delta = O(1)$, separable, spanning, K_n Ferber, Lee, Mousset, 16
- $\Delta = O(1)$, spanning, multipart., quasiran. Kim, Kühn, Osthus, Tyomkyn, 19

Graph decompositions - progress

Approximate decompositions:

- $\Delta = O(1)$, trees, almost spanning, K_n Böttcher, Hladký, Piguet, Taraz, 16
- $\Delta = O(1)$, separable, almost spanning, K_n Messuti, Rödl, Schacht, 17
- $\Delta = O(1)$, separable, spanning, K_n Ferber, Lee, Mousset, 16
- $\Delta = O(1)$, spanning, multipart., quasiran. Kim, Kühn, Osthus, Tyomkyn, 19
- $\Delta \leq \text{poly}\, pn$, trees, spanning, $G_{n,p}$ Ferber, Samotij, 18^+

Graph decompositions - progress

Approximate decompositions:

- $\Delta = O(1)$, trees, almost spanning, K_n Böttcher, Hladký, Piguet, Taraz, 16
- $\Delta = O(1)$, separable, almost spanning, K_n Messuti, Rödl, Schacht, 17
- $\Delta = O(1)$, separable, spanning, K_n Ferber, Lee, Mousset, 16
- $\Delta = O(1)$, spanning, multipart., quasiran. Kim, Kühn, Osthus, Tyomkyn, 19
- $\Delta \leq \operatorname{poly} pn$, trees, spanning, $G_{n,p}$ Ferber, Samotij, 18^+
- $\Delta \leq n/\log n$, bo. dege., spanning, quasiran. Allen, Böttcher, Hladký, Piguet, 19

Graph decompositions - progress

Approximate decompositions:

- $\Delta = O(1)$, trees, almost spanning, K_n Böttcher, Hladký, Piguet, Taraz, 16
- $\Delta = O(1)$, separable, almost spanning, K_n Messuti, Rödl, Schacht, 17
- $\Delta = O(1)$, separable, spanning, K_n Ferber, Lee, Mousset, 16
- $\Delta = O(1)$, spanning, multipart., quasiran. Kim, Kühn, Osthus, Tyomkyn, 19
- $\Delta \leq \text{poly}\, pn$, trees, spanning, $G_{n,p}$ Ferber, Samotij, 18^+
- $\Delta \leq n/\log n$, bo. dege., spanning, quasiran. Allen, Böttcher, Hladký, Piguet, 19

Decompositions:

- $\Delta = O(1)$, trees, (almost) spanning, quasiran. J., Kim, Kühn, Osthus, 19

Graph decompositions - progress

Approximate decompositions:

- $\Delta = O(1)$, trees, almost spanning, K_n Böttcher, Hladký, Piguet, Taraz, 16
- $\Delta = O(1)$, separable, almost spanning, K_n Messuti, Rödl, Schacht, 17
- $\Delta = O(1)$, separable, spanning, K_n Ferber, Lee, Mousset, 16
- $\Delta = O(1)$, spanning, multipart., quasiran. Kim, Kühn, Osthus, Tyomkyn, 19
- $\Delta \leq \text{poly}\, pn$, trees, spanning, $G_{n,p}$ Ferber, Samotij, 18^+
- $\Delta \leq n/\log n$, bo. dege., spanning, quasiran. Allen, Böttcher, Hladký, Piguet, 19

Decompositions:

- $\Delta = O(1)$, trees, (almost) spanning, quasiran. J., Kim, Kühn, Osthus, 19
- Oberwolfach problem Glock, J., Kim, Kühn, Osthus, 18^+

Graph decompositions - progress

Approximate decompositions:

- $\Delta = O(1)$, trees, almost spanning, K_n Böttcher, Hladký, Piguet, Taraz, 16
- $\Delta = O(1)$, separable, almost spanning, K_n Messuti, Rödl, Schacht, 17
- $\Delta = O(1)$, separable, spanning, K_n Ferber, Lee, Mousset, 16
- $\Delta = O(1)$, spanning, multipart., quasiran. Kim, Kühn, Osthus, Tyomkyn, 19
- $\Delta \leq \text{poly}\, pn$, trees, spanning, $G_{n,p}$ Ferber, Samotij, 18^+
- $\Delta \leq n/\log n$, bo. dege., spanning, quasiran. Allen, Böttcher, Hladký, Piguet, 19

Decompositions:

- $\Delta = O(1)$, trees, (almost) spanning, quasiran. J., Kim, Kühn, Osthus, 19
- Oberwolfach problem Glock, J., Kim, Kühn, Osthus, 18^+
- as ABHP + many leaves Allen, Böttcher, Clemens, Taraz, 19^+

Graph decompositions - progress

Approximate decompositions:

- $\Delta = O(1)$, trees, almost spanning, K_n Böttcher, Hladký, Piguet, Taraz, 16
- $\Delta = O(1)$, separable, almost spanning, K_n Messuti, Rödl, Schacht, 17
- $\Delta = O(1)$, separable, spanning, K_n Ferber, Lee, Mousset, 16
- $\Delta = O(1)$, spanning, multipart., quasiran. Kim, Kühn, Osthus, Tyomkyn, 19
- $\Delta \leq \operatorname{poly} pn$, trees, spanning, $G_{n,p}$ Ferber, Samotij, 18^+
- $\Delta \leq n/\log n$, bo. dege., spanning, quasiran. Allen, Böttcher, Hladký, Piguet, 19

Decompositions:

- $\Delta = O(1)$, trees, (almost) spanning, quasiran. J., Kim, Kühn, Osthus, 19
- Oberwolfach problem Glock, J., Kim, Kühn, Osthus, 18^+
- as ABHP + many leaves Allen, Böttcher, Clemens, Taraz, 19^+
- Ringel's conjecture Pokrovskiy, Montgomery, Sudakov, 20^+

Graph decompositions - progress

Approximate decompositions:

- $\Delta = O(1)$, trees, almost spanning, K_n Böttcher, Hladký, Piguet, Taraz, 16
- $\Delta = O(1)$, separable, almost spanning, K_n Messuti, Rödl, Schacht, 17
- $\Delta = O(1)$, separable, spanning, K_n Ferber, Lee, Mousset, 16
- $\Delta = O(1)$, spanning, multipart., quasiran. Kim, Kühn, Osthus, Tyomkyn, 19
- $\Delta \leq \operatorname{poly} pn$, trees, spanning, $G_{n,p}$ Ferber, Samotij, 18^+
- $\Delta \leq n/\log n$, bo. dege., spanning, quasiran. Allen, Böttcher, Hladký, Piguet, 19

Decompositions:

- $\Delta = O(1)$, trees, (almost) spanning, quasiran. J., Kim, Kühn, Osthus, 19
- Oberwolfach problem Glock, J., Kim, Kühn, Osthus, 18^+
- as ABHP + many leaves Allen, Böttcher, Clemens, Taraz, 19^+
- Ringel's conjecture Pokrovskiy, Montgomery, Sudakov, 20^+
- 2-factors, quasiran. Keevash, Staden, 20^+

Graph decompositions - progress

Approximate decompositions:

- $\Delta = O(1)$, trees, almost spanning, K_n Böttcher, Hladký, Piguet, Taraz, 16
- $\Delta = O(1)$, separable, almost spanning, K_n Messuti, Rödl, Schacht, 17
- $\Delta = O(1)$, separable, spanning, K_n Ferber, Lee, Mousset, 16
- $\Delta = O(1)$, spanning, multipart., quasiran. Kim, Kühn, Osthus, Tyomkyn, 19
- $\Delta \leq \text{poly}\, pn$, trees, spanning, $G_{n,p}$ Ferber, Samotij, 18^+
- $\Delta \leq n/\log n$, bo. dege., spanning, quasiran. Allen, Böttcher, Hladký, Piguet, 19

Decompositions:

- $\Delta = O(1)$, trees, (almost) spanning, quasiran. J., Kim, Kühn, Osthus, 19
- Oberwolfach problem Glock, J., Kim, Kühn, Osthus, 18^+
- as ABHP + many leaves Allen, Böttcher, Clemens, Taraz, 19^+
- Ringel's conjecture Pokrovskiy, Montgomery, Sudakov, 20^+
- 2-factors, quasiran. Keevash, Staden, 20^+
- Ringel's conjecture, quasiran. Keevash, Staden, 20^+

Hypergraph decompositions - progress

Hypergraph decompositions - progress

Existence conjecture: Keevash, 14^{+}

Hypergraph decompositions - progress

Existence conjecture: Keevash, 14^+

Alternative proof + extensions: Glock, Kühn, Lo, Osthus, 17^+

Hypergraph decompositions - progress

Existence conjecture: Keevash, 14^+

Alternative proof + extensions: Glock, Kühn, Lo, Osthus, 17^+

Multipartite setting: Keevash, 18^+

Hypergraph decompositions - progress

Existence conjecture: Keevash, 14^+

Alternative proof + extensions: Glock, Kühn, Lo, Osthus, 17^+

Multipartite setting: Keevash, 18^+

Quasirandom hypergraphs:
$\epsilon > 0$, $t \in \mathbb{N}$, $d \in (0,1]$ and suppose G has n vertices.
G is (ϵ, t, d)-typical if

$$\left| \bigcap_{S \in \mathcal{S}} N_G(S) \right| = (1 \pm \epsilon) d^{|\mathcal{S}|} n$$

for all sets $\mathcal{S}$ of $(k-1)$-sets of $V(G)$ with $|\mathcal{S}| \le t$.

New results III

Theorem (Ehard, J., 2020^+)

$\forall k, \alpha, d_0 > 0 \ \exists \ n_0, t \in \mathbb{N}, \varepsilon > 0$:

New results III

Theorem (Ehard, J., 2020^+)

$\forall k, \alpha, d_0 > 0 \ \exists \ n_0, t \in \mathbb{N}, \varepsilon > 0$:
G k-graph, $n \geq n_0$ vertices, (ε, t, d)-typical with $d \geq d_0$

New results III

Theorem (Ehard, J., 2020^+)

$\forall k, \alpha, d_0 > 0 \; \exists \; n_0, t \in \mathbb{N}, \varepsilon > 0$:
G k-graph, $n \geq n_0$ vertices, (ε, t, d)-typical with $d \geq d_0$
$H_1, \ldots, H_\ell$ k-graphs, n vertices each,
$\Delta_1(H_i) \leq \alpha^{-1}$ and $\sum_{i \in [\ell]} e(H_i) \leq (1-\alpha)e(G)$.

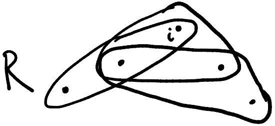
New results III

Theorem (Ehard, J., 2020$^+$)

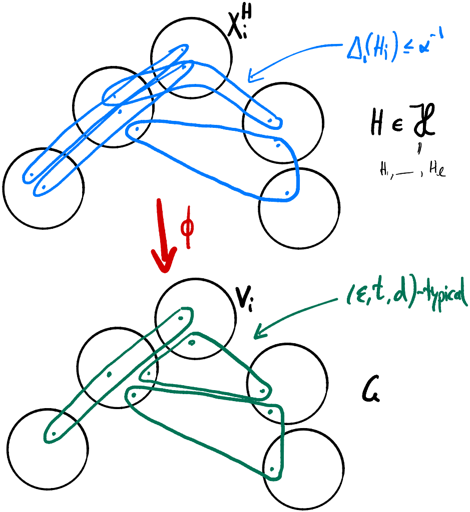
$\forall k, \alpha, d_0 > 0 \ \exists \ n_0, t \in \mathbb{N}, \varepsilon > 0$:
G k-graph, $n \geq n_0$ vertices, (ε, t, d)-typical with $d \geq d_0$
$H_1, \dots, H_\ell$ k-graphs, n vertices each,
$\Delta_1(H_i) \leq \alpha^{-1}$ and $\sum_{i \in [\ell]} e(H_i) \leq (1-\alpha)e(G)$.
Then G contains $H_1, \dots, H_\ell$ as edge-disjoint subgraphs.

Multipartite hypergraphs

k=3 (for simplicity)



- $V(R)=\{1,\dots,r\}$
- $\Delta_1(R)\le\alpha^{-1}$
- $|X_i|=|V_i|=(1\pm\varepsilon)n$
- $r\le n^{\log n}$
- $\sum_{H\in\mathcal{H}} e_H(X_{i_1}^H,\dots,X_{i_k}^H)\le(1-\alpha)dn^k$



New results IV

Theorem (Ehard, J., 2020^+)

Approx. decomp. of quasirandom multipartite k-graphs
into bounded degree k-graphs with the same multipartite structure.

New results IV

Theorem (Ehard, J., 2020^+)

Approx. decomp. of quasirandom multipartite k-graphs into bounded degree k-graphs with the same multipartite structure.

Hypergraph blow-up lemma for approximate decompositions for quasirandom k-graphs

New results IV

Theorem (Ehard, J., 2020^+)

Approx. decomp. of quasirandom multipartite k-graphs into bounded degree k-graphs with the same multipartite structure.

Hypergraph blow-up lemma for approximate decompositions for quasirandom k-graphs
Asked by Keevash and Kim, Kühn, Osthus, Tyomkyn

New results IV

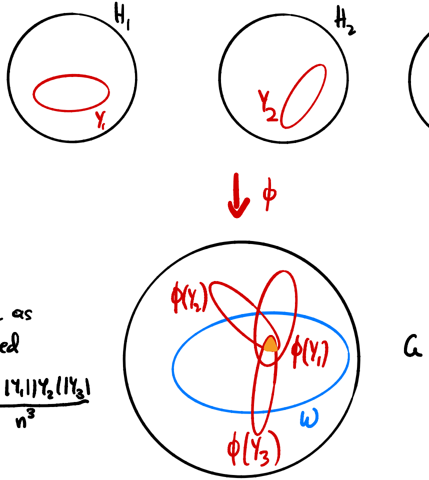
Theorem (Ehard, J., 2020^+)

Approx. decomp. of quasirandom multipartite k-graphs into bounded degree k-graphs with the same multipartite structure.

Hypergraph blow-up lemma for approximate decompositions for quasirandom k-graphs
Asked by Keevash and Kim, Kühn, Osthus, Tyomkyn

More features:
The packing itself exhibits strong quasirandom properties which is very useful for applications

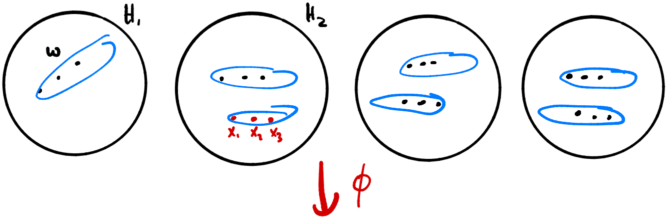
Quasirandom properties I



= as large as expected

$$= (1 \pm \alpha) \frac{|W||Y_1||Y_2||Y_3|}{n^3}$$

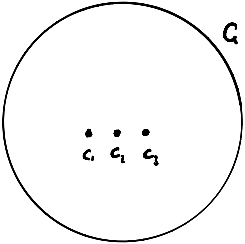
Quasirandom properties II



$$w : \bigcup_{H \in \mathcal{H}} V(H)^3 \to [0,1]$$

s.t. $w(x) > 0$

$\Rightarrow x \in e \in E(\mathcal{H})$



Construct ϕ such that

$$w\left(\phi^{-1}((c_1, c_2, c_3))\right) = (1 \pm \alpha)\frac{w(\text{total})}{n^3}$$

Proof ideas

Multipartite setting implies the other setting

Proof ideas

Multipartite setting implies the other setting

1. Proceed cluster by cluster:
iteratively embed almost all vertices in $\bigcup_{i \in [\ell]} X_j^{H_i}$ into V_j

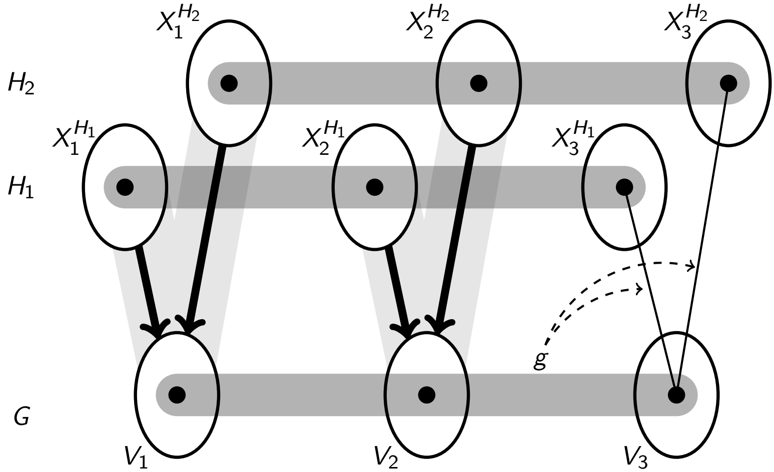
Proof ideas

Multipartite setting implies the other setting

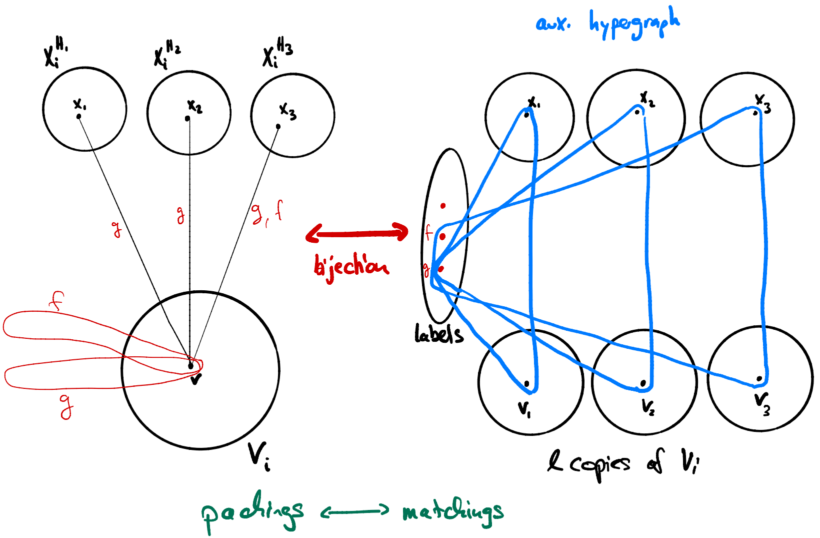
1. Proceed cluster by cluster:
iteratively embed almost all vertices in $\bigcup_{i\in[\ell]} X_j^{H_i}$ into V_j

2. Complete the embedding using an extra edge slice

Proof ideas II



Proof ideas III



Summary II

Approx. decompositions of quasirandom k-graphs
in the normal and multipartite setting
into bounded degree k-graphs

Applications

Consider a hypergraph as a simplicial complex:
Hamilton cycle in a k-graph = spanning $\mathbb{S}^{k-1}$

Applications

Consider a hypergraph as a simplicial complex:
Hamilton cycle in a k-graph $=$ spanning $\mathbb{S}^{k-1}$

Georgakopoulos, Haslegrave, Narayanan, Montgomery, 18^{+}:
3-graph G with $\delta(G) \geq (\frac{1}{3} + o(1))n$, then G contains a spanning $\mathbb{S}^2$

Applications

Consider a hypergraph as a simplicial complex:
Hamilton cycle in a k-graph = spanning $\mathbb{S}^{k-1}$

Georgakopoulos, Haslegrave, Narayanan, Montgomery, 18$^+$:
3-graph G with $\delta(G) \geq (\frac{1}{3} + o(1))n$, then G contains a spanning $\mathbb{S}^2$

Ehard, J. 20$^+$:
Typical 3-graphs can be approx. decomposed into spanning $\mathbb{S}^2$

Applications

Consider a hypergraph as a simplicial complex:
Hamilton cycle in a k-graph = spanning $\mathbb{S}^{k-1}$

Georgakopoulos, Haslegrave, Narayanan, Montgomery, 18^+:
3-graph G with $\delta(G) \geq (\frac{1}{3} + o(1))n$, then G contains a spanning $\mathbb{S}^2$

Ehard, J. 20^+:
Typical k-graphs can be approx. decomposed into spanning $\mathbb{S}^{k-1}$