Minimum pair-degree for Hamiltonian cycles in 4-graphs

Joint work with J. Polcyn, Chr. Reiher, V. Rödl, M. Schacht, and B. Schülke*

July 29, 2020

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1

[1] *Bjarne actually made these slides, so all errors are his... (smile)

Overview

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Theorem (Dirac 1952)

Let G be a graph on $n \geq 3$ vertices with $\delta(G) \geq \frac{1}{2}n$, then G contains a Hamiltonian cycle.

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- $k \geq 2$, *k-uniform hypergraph* (*k-graph*): Pair (V, E), with vertex set V and edge set $E \subseteq V^{(k)} := \{e \subseteq V : |e| = k\}$

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- (tight) path (of length $\ell - k + 1$): $\{x_1, \ldots, x_\ell\} \subset V$, every consecutive k-tuple of vertices $\{x_i, x_{i+1}, \ldots, x_{i+k-1}\}$ with $i \in [\ell - k + 1]$.

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- (tight) cycle (of length ℓ): $\{x_1, \ldots, x_\ell\} \subset V$, every consecutive k-tuple of vertices $\{x_i, x_{i+1}, \ldots, x_{i+k-1}\}$ with $i \in \mathbb{Z}/n\mathbb{Z}$

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- *$d_H(\{x, y\})$ is the pair-degree of a vertex pair $\{x, y\}$: the number of edges in H containing $\{x, y\}$.*
- *$\delta_2(H)$ is the minimum pair-degree of a hypergraph H: the smallest pair-degree taken over all vertex pairs in H.*

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Theorem (Rödl, R., Szemerédi (2008))

For every $k \geq 2$ and large n, every n-vertex k-graph H with $\delta_{k-1}(H) \geq \left(\frac{1}{2} + o(1)\right) n$ contains a Hamiltonian cycle.

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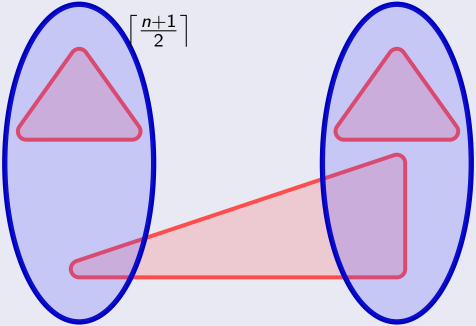

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For large n, every 3-graph H satisfying $\delta_1(H) \geq \left(\frac{5}{9} + o(1)\right) \frac{n^2}{2}$ contains a Hamiltonian cycle.

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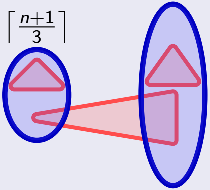

Main theorem

Theorem (Polcyn, Reiher, Rödl, R., Schacht, Schülke (2020))

For every $\alpha > 0$, there exists an integer n_0 such that every 4-uniform hypergraph H with $n \geq n_0$ vertices and minimum pair-degree $\delta_2(H) \geq \left(\frac{5}{9} + \alpha\right)\frac{n^2}{2}$ contains a Hamiltonian cycle.

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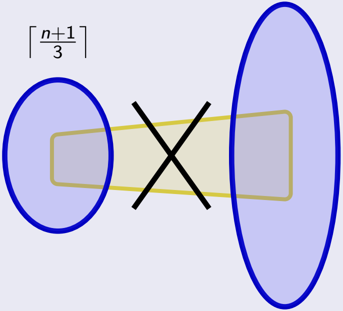

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Conjecture

For all $k \geq 3$ and large n, any k-graph H on n vertices with $\delta_{k-2}(H) \geq \left(\frac{5}{9} + o(1)\right)\frac{n^2}{2}$ contains a Hamiltonian cycle.

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Remark

Proved this summer by Polcyn, Reiher, Rödl, and Schülke, and, independently, by R. Lang and N. Sanhueza-Matamala.

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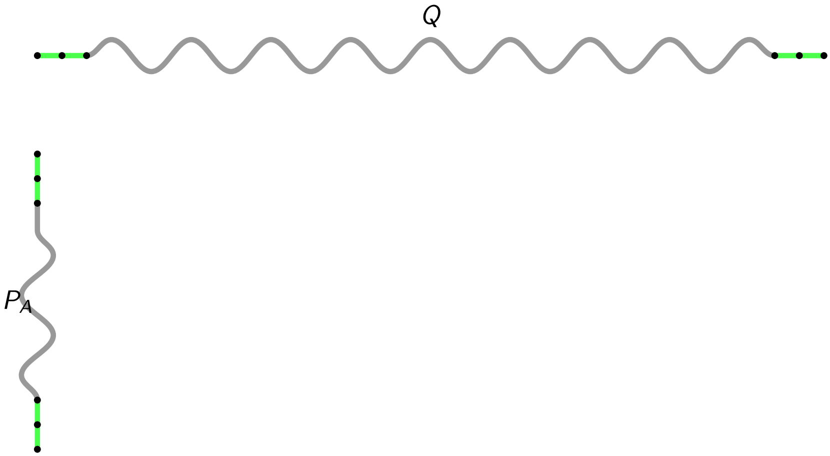

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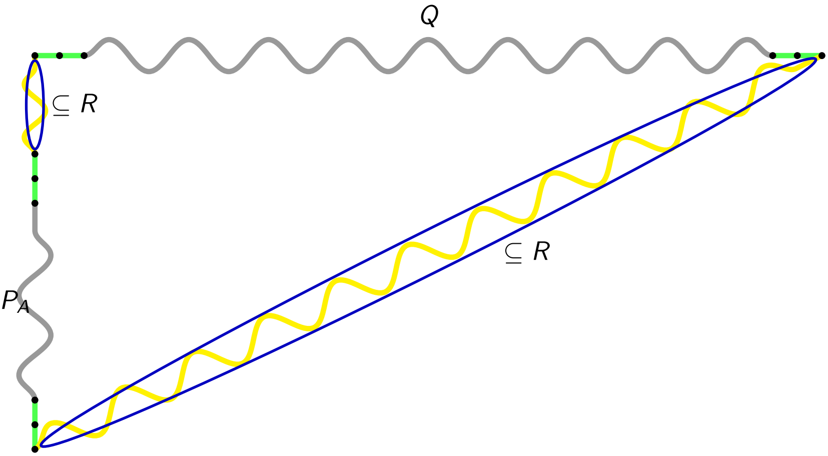

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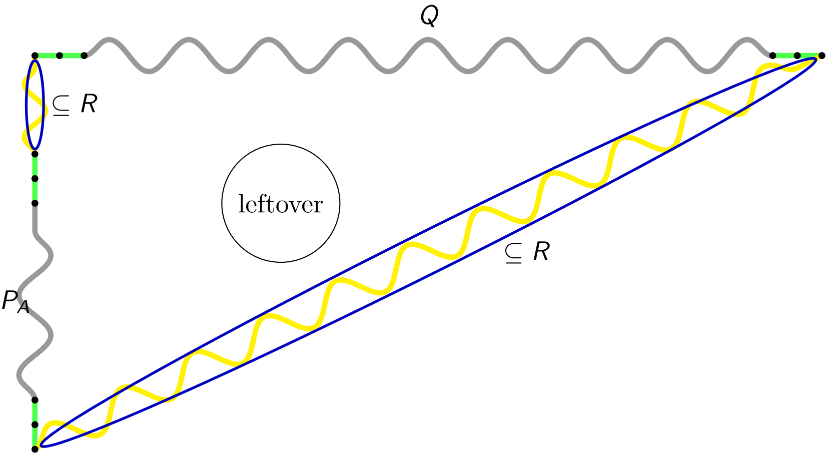

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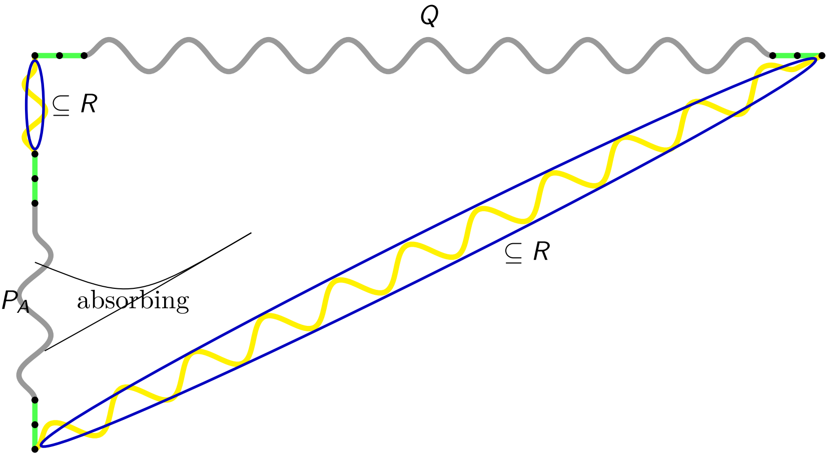

Connecting Lemma

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Definition

A 3-tuple $(x, y, z) \in V^3$ is called ζ-*connectable* in H if the set

$$U_{xyz} = \{v \in V : xyz \in H_v \text{ and } xy, yz \text{ are "}\zeta\text{-connectable" in } H_v\}$$

has size $|U_{xyz}| \geq \zeta n$.

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Lemma (Connecting Lemma)

There is an integer L such that the following holds. If (a, b, c) and (x, y, z) are disjoint, connectable triples, then there are $\Omega(n^L)$ $abc - xyz$-paths in H with L inner vertices.

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- Let S^* be the set of sextuples $(d_1, d_2, u, v, d_3, d_4)$ such that:

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- Can show that $|S^*| = \Omega(n^6)$

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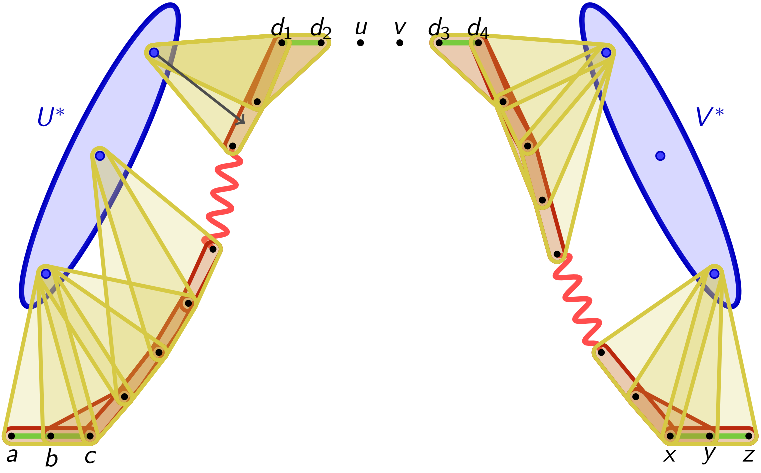
$u \in U_{abc}$, $v \in U_{xyz}$, (d_i) 4-path in H_{uv}, d_1, d_2 conn. in H_u, d_3, d_4 conn. in H_v



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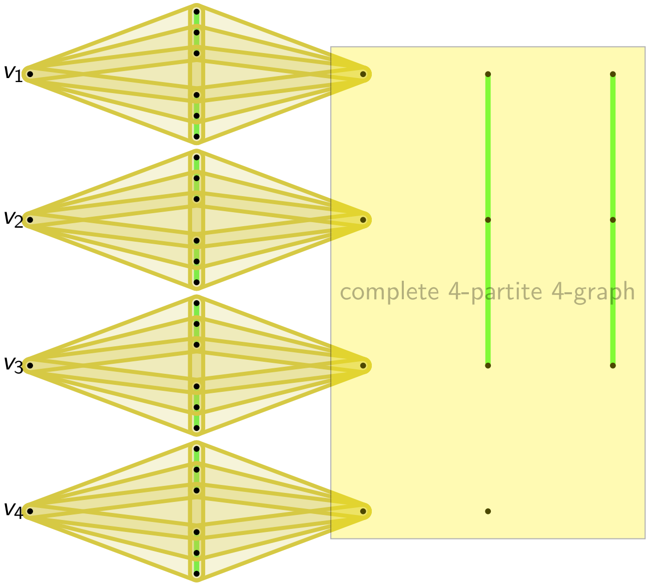
Remark

The divisibility condition in (*iii*) can be dealt with easily by adjusting the Connecting Lemma.

Absorbers



Proof sketch Absorbing Path

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  - (iii) $|\mathcal{A}|$ is small.
- Connect all elements of $\mathcal{A}$ into an absorbing path P_A

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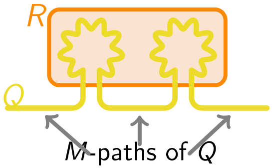
Remark

The proof doesn't rely on Szemerédi's regularity lemma.

Proof Idea Covering Lemma

- Consider a longest path Q with this structure (or, a largest family $\mathcal{C}$ of disjoint M-paths):



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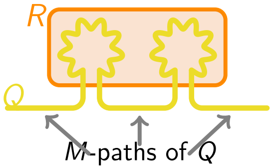

- Call M-vertex sets *blocks* and 'chop off' the set U of leftover vertices into blocks (leaving, possibly a remainder); there are altogether $|\mathcal{C}| + \lfloor |U|/M \rfloor$ blocks

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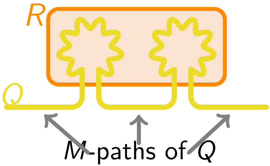

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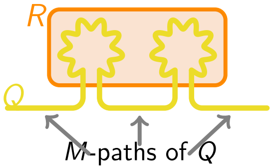

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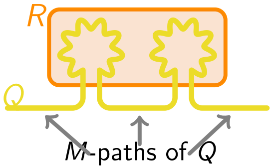

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- Random argument (weighted Janson ineq): there is a society that is useful for many $u \in U$

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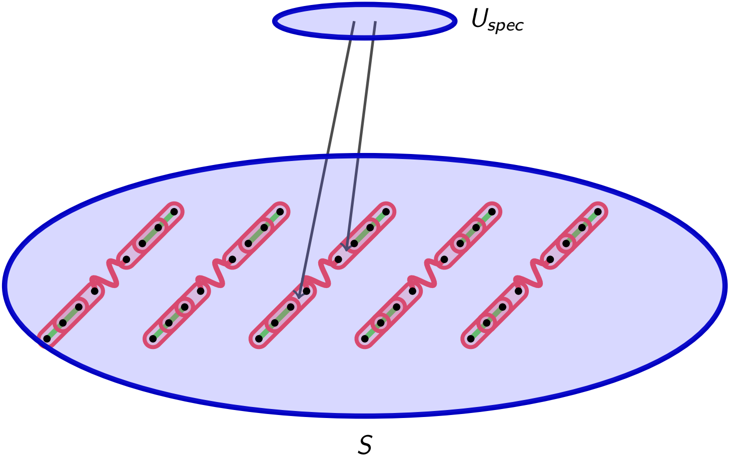

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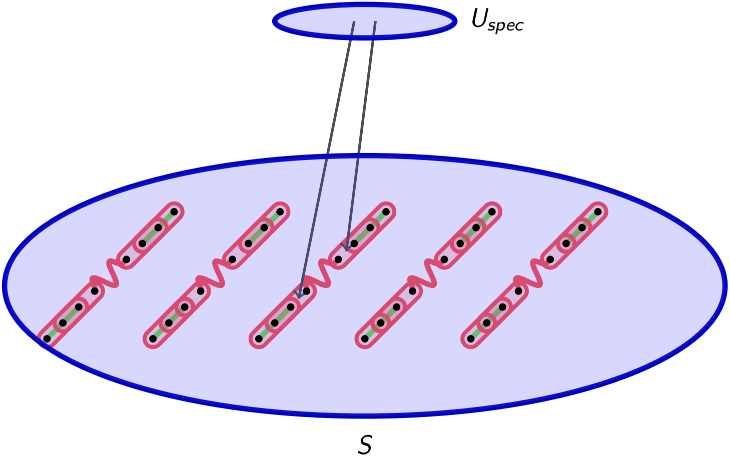

Creates $m+1$ new M-paths (from $\frac{3}{4}(M+1)$-paths by inserting $(M-3)/4$ vts of U);

Proof Idea Covering Lemma

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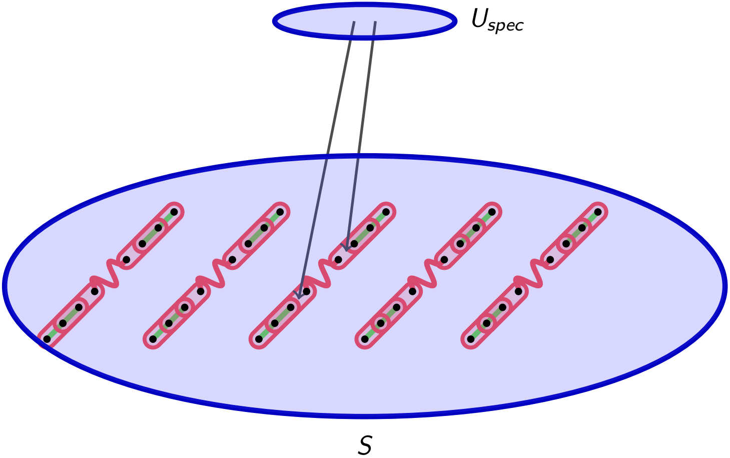

Creates $m+1$ new M-paths (from $\frac{3}{4}(M+1)$-paths by inserting $(M-3)/4$ vts of U); replacing in $\mathcal{C}$ the m paths from $\mathcal{S}$ by the new ones, yields a larger family $\mathcal{C}'$ – a contradiction !!!

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Given integers $k > d > 1$, determine the infimal real number α_d^k such that every k-uniform hypergraph H with minimum d-degree $\delta_d(H) \geq \left(\alpha_d^k + o(1)\right)\frac{n^{k-d}}{(k-d)!}$ contains a Hamiltonian cycle.

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Question

Is $\beta_3 = 5/8$?

THANK YOU FOR YOUR ATTENTION !!!